Elementary Charge Transfer Processes in a Superconductor-Ferromagnet Entangler

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We study the production of spatially separated entangled electrons in ferromagnetic leads from Cooper pairs in a superconducting lead. We give a complete description of the elementary charge transfer processes, i) transfer of Cooper pairs out of the superconductor by Andreev reflection and ii) distribution of the entangled quasiparticles among the ferromagnetic leads, in terms of their statistics. The probabilities that entangled electrons flow into spatially separated leads are completely determined by experimentally measurable conductances and polarizations. Finally, we investigate how currents, noise and cross correlations are affected by transport of entangled electrons.

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A solid state entangler is an electronic analog of the optical setups used for experimental Bell inequality tests, quantum cryptography and quantum teleportation [1]. Ideally, such a device should produce separated currents of entangled electrons. Superconductors are suitable candidates as sources in solid state entanglers since Cooper pairs constitute entangled states. This prospect has motivated several papers addressing the properties of hybrid superconductor and normal metal entanglers [2, 3, 4, 5]. One of the challenges is to prevent processes where pairs of entangled particles reach the same lead, *i.e.* are not spatially separated. Electrons from Cooper pairs are entangled in spin and energy space, and separation of pairs into different leads using ferromagnets or quantum dots has been suggested [3]. Upon filtering, only the spin or energy part of the two-particle wave function collapses, depending on whether ferromagnets or quantum dots are used. Respectively, energy or spin entanglement remains [4]. Here we consider separation by ferromagnets.

Solid state entanglers have been analyzed in Refs. [2, 3, 4, 5] in terms of currents, noise and cross correlations. A more direct approach, describing the elementary charge transfer processes in terms of experimentally controllable parameters is certainly desirable. We demonstrate how this is possible through the full distribution of current fluctuations, the full counting statistics (FCS), of the solid state entangler [6, 7, 8]. The FCS provides complete information about currents, noise, cross correlations and higher cumulants, and even more importantly, allows direct access to the probability for transfer of charge between different parts of the device.

We consider the singlet superconductor-ferromagnet (S-F) device shown in Fig. 1. A normal metal cavity (c) is connected to one superconducting terminal and several ferromagnetic terminals via tunnel junctions. The cavity is under the influence of proximity effect. In this device, charge transport occurs via two processes: i) Transfer of Cooper pairs out of the superconductor by Andreev reflection and ii) distribution of the entangled quasiparticles among the ferromagnetic leads. The distribution can occur via Direct Andreev (DA) reflection, where a entangled pair is transferred into lead F_n or crossed Andreev reflection (CA), where each particle of the entangled pair is transferred into spatially separated leads F_m and F_n ($n \neq m$). CA produces spatially separated entangled electrons. Since the ferromagnetic terminals are at the same voltage and we consider zero temperature, there is no direct electron transport between the ferromagnetic terminals [9].

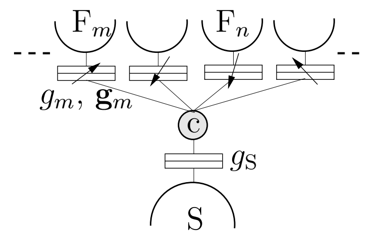

FIG. 1: Circuit theory representation of generic F-S entangler. Entangled electrons from the singlet superconductor S enter the cavity c through a tunnel barrier with conductance g_S and escape through ferromagnetic interfaces with conductance g_n and spin polarization $|\mathbf{g}_n|/g_n$ into drains F_n. Arrows indicate magnetization directions $\mathbf{g}_n/|\mathbf{g}_n|$.

Our general results for the counting statistics show that the processes i) and ii) are independent and therefore the statistics can be factorized. This novel factorization and the probability distribution for process ii) reveals the precise dependence of the probabilities for CA and DA processes on the experimentally measurable conductances and polarizations of the ferromagnetic leads. We find that the probability that the electrons of a Cooper pair transferred into the cavity are detected in terminals m and n is

$$p_{mn} = (g_m g_n - \mathbf{g}_m \mathbf{g}_n) / (g^2 - \mathbf{g}^2), \tag{1}$$

where $g = \sum_n g_n$ and $\mathbf{g} = \sum_n \mathbf{g}_n$. The probabilities p_{mn} depend solely on the conductances g_n and spin polariza-

tion conductances $\mathbf{g}_n$ of the ferromagnetic leads which can be determined by magnetoresistance measurement in the normal state. Eq. (1) shows that the detected two-particle processes originate from a *pure spin singlet* density matrix subspace [10]. We emphasize that (1) in combination with the Cooper pair transfer probability, to be discussed below, allows for an unambiguous identification of all statistical properties of the charge transfer. Using the magnetization dependence of the probabilities one can violate Bell's inequality straightforwardly and demonstrate entanglement. In a device with *e.g.* two ferromagnetic terminals, the probability to separate the entangled quasi-particle pair into different leads is enhanced in an antiparallel magnetization configuration.

It is quite remarkable that all statistics (noise, cross correlations, and higher order cumulants) in this device are completely determined by the *normal state* transport conductances between the cavity and the terminals and no additional parameters need to be introduced. The relative orientations of the magnetizations are the control parameters in an experimental situation. The fraction of the CA current and, therefore, the spatially separated entangled pair currents follow from these.

Ferromagnet detection of entangled spin singlets from a ballistic normal conductor was considered in Ref. [10]. In that device, there are also one-particle transfers, which can contribute substantially to the current and the noise. Another important qualitative difference between our device and the system in Ref. [10] is the latter's strong coupling to the detectors which can distort the spin singlets emitted from the source and induce triplet correlations upon detection. Also, in the limit of a weak coupling to the detectors, there are no two-particle processes in the system of Ref. [10]. In contrast, the electrons in our S-F device are always detected from a pure spin singlet state.

The charge transfer probabilities are obtained by identifying the elementary processes in the many-body charge counting statistics. The statistics is determined by the cumulant generating function (CGF) $S(\chi_1, \ldots, \chi_N) = S(\{\chi_n\})$ of the probability $P(\{N_n\})$ to transfer in a time interval t_0, N_1 electrons to F_1, N_2 electrons to F_2, and so on. Our main finding is the statistics

$$P(\{N_n\}) \equiv \int \frac{d^n\chi}{(2\pi)^n} \mathrm{e}^{S(\{\chi_n\}) - i\sum_n \chi_n N_n} \tag{2a}$$

$$= P_{\mathrm{S}}\left(\sum\nolimits_n N_n\right) P\left(\{N_n\}\Big|\sum\nolimits_n N_n\right) \tag{2b}$$

for $\sum_n N_n$ even and positive. The interpretation of this result is that the charge transfer is given by two independent processes. The first factor $P_S(2N)$ is the probability that $N = \sum_n N_n/2$ Cooper pairs are emitted from the superconducting source terminal into any of the detectors. The second factor $P(\{N_n\}|2N)$ in (2b) is the conditional probability that N_n out of the $2N$ electrons have been transferred into ferromagnetic terminal F_n. Below we will explain in detail how our calculation yields concrete expressions for the elementary processes described by $P(\{N_n\}|2N)$. These results facilitates a unique interpretation of the transfer of spin singlet electron pairs.

To complete the full statistical description, we now supply the microscopic expressions for the two probabilities in (2b). The Cooper pair transfer probability is obtained from $P_{\mathrm{S}}(2N) = \int d\chi/(2\pi)\exp(S_{\mathrm{S}}(\chi) - iN\chi)$ with a CGF $S_{\mathrm{S}}(\chi)$ given by

$$\frac{t_0 V}{\sqrt{2}e}\sqrt{g_\Sigma^2 + \sqrt{(g_{\mathrm{S}}^2 - g^2 + \mathbf{g}^2)^2 + 4g_{\mathrm{S}}^2(g^2 - \mathbf{g}^2)e^{2i\chi}}}\,, \tag{3}$$

where $g_\Sigma^2 = g_{\mathrm{S}}^2 + g^2 + \mathbf{g}^2$. The contact to the superconducting terminal is characterized by a spin-independent conductance g_{S}. The π-periodicity of $S_{\mathrm{S}}(\chi)$ on χ ensures that an even number of charges is transferred. The $2N$ electrons are distributed among the F_n terminals according to the multinomial distribution $P(\{N_n\}|2N) = \int d^n\chi/(2\pi)^n \exp(S_{\mathrm{N}}(\{\chi_n\}) - i\sum_n \chi_n N_n)$ with a CGF

$$S_{\mathrm{N}}(\{\chi_n\}) = N \ln\left(\sum\nolimits_{mn} p_{mn} e^{-i\chi_m - i\chi_n}\right). \tag{4}$$

The concrete form of the two-particle probabilities p_{mn} to detect one charge in terminal m and one in terminal n is given in (1). We will explain below how our calculation based on the circuit theory of full counting statistics determines Eqs.(1),(3), and (4).

We emphasize that our interpretation in terms of two independent processes of charge transfer is based on a detailed calculation with a general result for the counting statistics. We have not made any a priori assumptions on the initial state of the superconducting source terminal or the ferromagnetic terminals, except that they are reservoirs at zero temperature with a voltage bias eV applied. The *direct result* of our calculation is the CGF $S(\{\chi_n\})$ of the S-F entangler of Fig. 1. Its explicit expression is found in (3) by replacing the factor $\mathrm{e}^{2i\chi}$ with $\exp\{S_{\mathrm{N}}(\{\chi_n\})/N\}$ given in (4). The factorization in (2) can be proven straightforwardly from $S(\{\chi_n\})$. Actually, such a factorization is valid for any CGF where the χ dependence is $\exp\{S_{\mathrm{N}}(\{\chi_n\})/N\}$, irrespectively of its form or the probabilities p_{mn}.

We now discuss some consequences of the charge counting statistics. FCS enables us to express the current and noise correlations in a compact and meaningful form. The currents $I_n = (ie/t_0)\partial S(\{\chi_n\})/\partial\chi_n|_{\chi_n=0}$ are

$$I = GV,\ G = \frac{g_{\mathrm{S}}^2(g^2 - \mathbf{g}^2)}{\sqrt{g_{\mathrm{S}}^2 + g^2}(g_{\mathrm{S}}^2 + g^2 - \mathbf{g}^2)},\ I_n = Ip_n, \tag{5}$$

where $p_n = \sum_m p_{mn}$ is the probability to detect one of the electrons in terminal n, irrespective where the second electron goes. The combined probabilities can be directly accessed in the noise correlators between current fluctuations in terminals m and n, $C_{mn} = (-2e^2/t_0)\partial^2 S(\{\chi_n\})/\partial\chi_m\partial\chi_n|_{\chi_{n,m}=0}$:

$$C_{mn} = 2eI\left[p_{mn} + p_n\delta_{mn} - 2(1 - F_2)p_m p_n\right]. \tag{6}$$

The Fano factor for Cooper pair transport is defined as the ratio of the full current noise $C = \sum_{mn} C_{mn}$ to the Poissonian noise of doubled charges, $F_2 = C/4eI$, and is explicitly found $2(1-F_2) = [5 - \mathbf{g}^2/(g_S^2 + g^2)]x/(1+x)^2$ where $x = g_S^2/(g^2 - \mathbf{g}^2)$. These expressions for the current and the noise provide a transparent interpretation of the transport processes. The current in (5) is proportional to $g_S^2(g^2 - \mathbf{g}^2)$, since two particles have to tunnel through the double junction to transfer a Cooper pair from S. The denominator is due to the proximity effect [11, 12, 13] and enhances the current drastically in comparison to calculations based on the tunneling Hamiltonian [9]. The current into each terminal I_n is then weighted according to the probability p_n. We might also distinguish the contributions to the current originating from crossed and direct Andreev reflection. The probability to detect a DA reflection in terminal n is given by p_{nn}, and the probability for CA detection in different terminals $m \neq n$ is given by p_{mn}. We find the ratio of the crossed current to the total current as

$$\frac{I_n^{\mathrm{CA}}}{I_n} = \frac{p_n - p_{nn}}{p_n} = \frac{g_n(g - g_n) - \mathbf{g}_n(\mathbf{g} - \mathbf{g}_n)}{g_n g - \mathbf{g}_n \mathbf{g}} . \qquad (7)$$

This ratio is independent of the coupling to the superconducting terminal. We further observe that the crossed current is enhanced by increasing the polarization of the contact n and is additionally favored by aligning the magnetization $\mathbf{g}_n$ opposite to the average magnetization $\mathbf{g}$. These results are a direct consequence of the spin-singlet nature of the Cooper pairs. Enhancing the magnitude of the polarization $|\mathbf{g}_n|/g_n$ of one terminal reduces the total current, but enhances the crossed part of the Andreev current, since the tunneling of one spin-singlet electron-hole pair through the same contact is strongly suppressed.

The sign of cross correlations in three-terminal beam splitters has been considered for various devices both experimentally [14] and theoretically [5, 15]. Studies of noise [16] and FCS [17] for a beam splitter with entangled electrons show that entanglement gives qualitatively different noise characteristics compared to transport of non-entangled electrons. The physical origin of positive and negative contributions to the cross correlators can in our case be understood from the dependence on the two-particle probabilities in (6). CA reflection leads to positive cross correlations since two particles are transferred simultaneously into F_m and F_n (bunching behavior) [16]. A negative contribution (anti bunching) that does not depend on entanglement, is induced by the fermion exclusion principle: The transfer of one electron-hole pair into F_m by DA reflection, prevents the simultaneous transfer of another pair into F_n. However, if the electron-hole pair transfers are not temporally correlated (Poissonian statistics), the exclusion principle does not affect the cross correlations. This is the case when there is strong asymmetry in the junction conductances g and g_S ($g \gg g_S$ or $g \ll g_S$) so that the Fano factor $F_2 = 1$. In this limit the negative contribution $-2(1-F_2)p_m p_n$ in (6) vanishes. Scattering matrix calculations give similar results for C_{mn} [18].

The strongly asymmetric case is particularly interesting since the cross correlations ($m \neq n$) $C_{mn} = 2eIp_{mn}$, are a *direct* measure of the probability that electrons from a Cooper pair are transferred into different terminals.

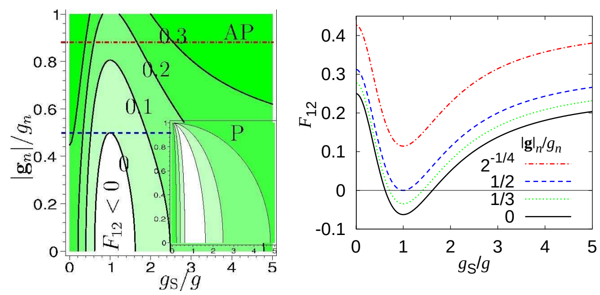

FIG. 2: (color online) Left: Contour plots of F_{12} for antiparallel and parallel (inset) configuration ($|\mathbf{g}_1| = |\mathbf{g}_2|$). Positive regions in green. Right: Plots of F_{12} in antiparallel configuration as a function of the conductance asymmetry g_S/g, legend denotes the value of the polarization $|\mathbf{g}_n|/g_n$. Red and blue horizontal lines in left panel correspond to red and blue curves in right panel.

To illustrate our theory, let us now consider the three-terminal version of Fig. 1 with the superconducting source terminal S and two ferromagnetic drains F_1 and F_2. The ferromagnetic magnetizations can in this device be utilized as filters to produce currents of entangled electrons in separated leads. Let us consider $|\mathbf{g}_1| = |\mathbf{g}_2|$ in the following and define Fano factors $F_{mn} = C_{mn}/(2eI)$. The autocorrelation noise $F_{11(22)}$ will be reduced in antiparallel alignment $\mathbf{g}_1 = -\mathbf{g}_2$ as compared to a S-N system ($\mathbf{g}_n = 0$) due to enhancement of CA. The cross correlation F_{12}, shown in Fig. 2 can have both positive and negative sign depending on the conductance asymmetry g_S/g and the spin polarization. The positive contribution to F_{12} is proportional to $g_1 g_2 + (-)|\mathbf{g}_1||\mathbf{g}_2|$ in the parallel (antiparallel) alignment demonstrating how spin filtering of entangled pairs enhances (reduces) the correlation between currents in F_1 and F_2 with respect to an S-N system [5]. Note that for sufficiently large spin polarization, F_{12} can be positive for the entire range of g_S/g in the antiparallel alignment (region above blue line in left panel of Fig. 2), whereas it remains always negative in the parallel alignment for $g_S/g \simeq 1$ (inset of Fig. 2). The change of sign in F_{12} by switching from antiparallel to parallel alignment is due to the enhanced probability of CA events, see (7).

We will finally outline the calculation that yields the FCS of the considered devices. We utilize the circuit theory of mesoscopic superconductivity [7, 19] and represent the circuit in terms of terminals, cavities and con-

nectors. Terminals are described by equilibrium quasiclassical Green's function matrices $\check{G}_n$ determined by electrochemical potential and temperature. Our notation for matrix subspace is: $\bar{}$ for spin, $\hat{}$ for Nambu, and $\check{}$ for Keldysh. Pauli matrices are denoted τ_j. At zero temperature we consider $0 < E \le eV$ where the Green's functions for all ferromagnetic terminals F_n are $\check{G} = \hat{\tau}_3\check{\tau}_3 + (\check{\tau}_1 + i\check{\tau}_2)$ where V is the voltage of the ferromagnetic terminals and E the quasiparticle energy. The superconductor S is at zero voltage and has Green's function $\check{G}_\mathrm{S} = \hat{\tau}_1$, where we assume $E \ll \Delta$, Δ being the gap of S. The terminals are connected to a cavity c which is under the influence of proximity effect from S. The cavity is described by an unknown Green's function $\check{G}_\mathrm{c}$, assumed isotropic due to chaotic or diffusive scattering. We assume that c is large enough so that charging effects can be neglected, and small enough so that $\check{G}_\mathrm{c}$ is spatially homogeneous. The circuit theory is formulated in terms of generalized matrix currents $\check{I}_j$ in spin$\otimes$Nambu$\otimes$Keldysh matrix space and from the matrix current conservation $\sum_j \check{I}_j = 0$. This determines the Greens function on the node together with the normalization condition $\check{G}_c^2 = 1$. The matrix currents can have arbitrary structure, and allow to derive the FCS by introducing the counting fields χ_n for each terminal according to [7] $\check{G}_n(\chi) = \mathrm{e}^{i\chi_n\hat{\tau}_3\check{\tau}_1/2}\check{G}\mathrm{e}^{-i\chi_n\hat{\tau}_3\check{\tau}_1/2}$. Spin active connectors are taken into account by spin dependent transmission and reflection amplitudes $t^n_{k,\sigma}$ and $r^n_{k,\sigma}$ for particles incident on the interface n from the cavity side in channel k with spin σ. The matrix current through a spin active tunnel barrier between c and F_n evaluated at the cavity side is [20, 21] $\check{I}_n = \left[g_n\check{G}_n/2 + \left\{\mathbf{g}_n\bar{\boldsymbol{\tau}}\hat{\tau}_3, \check{G}_n\right\}/4, \check{G}_\mathrm{c}\right]$. Here, $g_n = g_\mathrm{Q}\sum_{k,\sigma}|t^n_{k,\sigma}|^2$ is the tunnel conductance and $g_\mathrm{Q} = e^2/h$ the conductance quantum. The magnetization direction is encoded in the direction of $\mathbf{g}_n$, and the conductance polarization in that quantization axis is $|\mathbf{g}_n| = g_\mathrm{Q}\sum_k(|t^n_{k,\uparrow}|^2 - |t^n_{k,\downarrow}|^2)$. We have neglected here an additional term related to spin dependent phase shifts upon reflection at the interface [20, 21], as these are small in some material combinations or can be suppressed by a thin, non-magnetic oxide layer [22]. The matrix current between c and S is $\check{I}_\mathrm{S} = g_\mathrm{S}[\check{G}_\mathrm{S}, \check{G}_\mathrm{c}]/2$ [19]. We take into account the spin structure of matrix currents $\check{I}_n$ and Green's functions in S-F-systems, and derive the CGF in the linear response regime and for $eV \ll \Delta$, generalizing Ref. [15]: $S = t_0/(4e^2)\int \mathrm{d}E\sum_p\sqrt{\lambda_p^2}$, where $\{\lambda_p\}$ is the set of eigenvalues of the matrix $\check{M}$ defined by writing matrix current conservation in the cavity $\sum_j \check{I}_j \equiv [\check{M}, \check{G}_\mathrm{c}] = 0$. The non-trivial spin matrix structure of $\check{I}_n$ determines the magnetization dependence of transport processes in the system. Carrying out this procedure yields the FCS for the setup in Fig. 1.

In conclusion, we have investigated the elementary charge transfer processes of a S-F entangler. Charge transfers occurs via two statistically independent processes, i) Cooper pairs are transferred out of the superconductor by Andreev reflection and ii) entangled quasiparticles are distributed among the different ferromagnetic leads. The probabilities for entangled electrons to flow into spatially separated leads are completely determined by experimentally measurable conductances and polarizations. This allows complete knowledge of the statistics of charge transfer in the S-F entangler.

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[1] W. Tittel and G. Weihs, Quantum Inf. Comput. **1**, 3 (2001), and references therein.
[2] G. B. Lesovik, T. Martin, and G. Blatter, Eur. Phys. J. B **24**, 287 (2001); P. Samuelsson and M. Büttiker, Phys. Rev. Lett. **89**, 046601 (2002); P. Recher and D. Loss, Phys. Rev. Lett. **91**, 267003 (2003); P. Samuelsson, E. V. Sukhorukov, and M. Büttiker, Phys. Rev. Lett. **91**, 157002 (2003); L. Faoro, F. Taddei, and R. Fazio, Phys. Rev. B **69**, 125326 (2004); P. Samuelsson, E. V. Sukhorukov, and M. Büttiker, Phys. Rev. B **70**, 115330 (2004).
[3] P. Recher, E. V. Sukhorukov, and D. Loss, Phys. Rev. B **63**, 165314 (2001); N. M. Chtchelkatchev, G. Blatter, G. B. Lesovik, and T. Martin, Phys. Rev. B **66**, 161320(R) (2002).
[4] K. V. Bayandin, G. B. Lesovik, and T. Martin, Phys Rev. B **74**, 085326 (2006); Z. Y. Zeng, L. Zhou, J. Hong, and B. Li, Phys. Rev. B **74**, 085312 (2006).
[5] D. Sanchez, R. Lopez, P. Samuelsson, and M. Büttiker, Phys. Rev. B **68**, 214501 (2003); G. Bignon, M. Houzet, F. Pistolesi, and F. W. J. Hekking, Eur. Phys. J. B **67**, 110 (2004).
[6] L. S. Levitov and G. B. Lesovik, JETP Lett. **58**, 230 (1993).
[7] W. Belzig and Yu. V. Nazarov, Phys. Rev. Lett. **87**, 067006 (2001); W. Belzig and Yu. V. Nazarov, Phys. Rev. Lett. **87**, 197006 (2001).
[8] Yu. V. Nazarov, ed., *Quantum noise in mesoscopic physics* (Kluwer Academic Publishers, Dordrecht, 2003).
[9] G. Falci, D. Feinberg, and F. W. J. Hekking, Europhys. Lett. **54**, 255 (2001).
[10] A. Di Lorenzo and Yu. V. Nazarov, Phys. Rev. Lett. **94**, 210601 (2005).
[11] A. F. Volkov, A. V. Zaitsev, and T. M. Klapwijk, Physica C **210**, 21 (1993).
[12] Yu. V. Nazarov, Phys. Rev. Lett. **73**, 1420 (1994).
[13] J. P. Morten, A. Brataas, and W. Belzig, Phys. Rev. B **74**, 214510 (2006).
[14] S. Oberholzer, E. Bieri, C. Schonenberger, M. Giovan-

nini, and J. Faist, Phys. Rev. Lett. **96**, 046804 (2006).
[15] J. Börlin, W. Belzig, and C. Bruder, Phys. Rev. Lett. **88**, 197001 (2002).
[16] G. Burkard, D. Loss, and E. V. Sukhorukov, Phys. Rev. B **61**, R16303 (2000).
[17] F. Taddei and R. Fazio, Phys. Rev. B **65**, 075317 (2002).
[18] J. Torrès and T. Martin, Eur. Phys. J. B **12**, 319 (1999).
[19] Yu. V. Nazarov, Superlatt. Microstruct. **25**, 1221 (1999).
[20] D. Huertas-Hernando, Yu. V. Nazarov, and W. Belzig, Phys. Rev. Lett. **88**, 047003 (2002) and cond-mat/0204116; D. Huertas-Hernando PhD thesis, TU Delft (2002).
[21] D. Huertas-Hernando and Yu. V. Nazarov, Eur. Phys. J. B **44**, 373 (2005); A. Cottet and W. Belzig, Phys. Rev. B **72**, 180503(R) (2005); V. Braude and Yu. V. Nazarov, Phys. Rev. Lett. **98**, 077003 (2007).
[22] P. M. Tedrow, J. E. Tkaczyk, and A. Kumar, Phys. Rev. Lett. **56**, 1746 (1986).